Q: (80) If $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$; $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + 3\hat{j} - 2\hat{k}$
find (i) $\vec{A} \cdot (\vec{B} \times \vec{C})$ (ii) $|\vec{A} \times (\vec{B} \times \vec{C})|$, (iii) $(\vec{A} \times \vec{B})(\vec{B} \cdot \vec{C})$.

Ans: (iii) $(\vec{A} \times \vec{B})(\vec{B} \cdot \vec{C})$

$$\vec{B} \cdot \vec{C} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + 3\hat{j} - 2\hat{k}) = 2 + 3 + 2 = 7.$$

Now
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}$$
$$= (2+3)\hat{i} - (-1+6)\hat{j} + (1+4)\hat{k}$$
$$= 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$\therefore$
$$(\vec{A} \times \vec{B})(\vec{B} \cdot \vec{C}) = (\vec{B} \cdot \vec{C})(\vec{A} \times \vec{B})$$
$$= 7(5\hat{i} - 5\hat{j} + 5\hat{k}) = 35\hat{i} - 35\hat{j} + 35\hat{k}$$

Ans:

Q: Find the volume of a parallelopiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$; $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$.

Ans:- We know that the volume of a parallelopiped is $|\vec{A} \cdot (\vec{B} \times \vec{C})|$; where $\vec{A}, \vec{B}, \vec{C}$ are the edges of the parallelopiped.

Now
$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$
$$= (4-1)\hat{i} - (2+3)\hat{j} + (-1-6)\hat{k}$$
$$= 3\hat{i} - 5\hat{j} - 7\hat{k}.$$

Required volume of the parallelopiped;
$$|\vec{A} \cdot (\vec{B} \times \vec{C})| = |(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 5\hat{j} - 7\hat{k})|$$
$$= |6 + 15 - 28| = |-7| = 7 \quad \text{Ans:}$$

Q-92: Find the constant α, such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\alpha\hat{j}+5\hat{k}$ are coplanner.

Ans:- Hints, when the vectors are coplanner, they will not form a Parallelopiped.

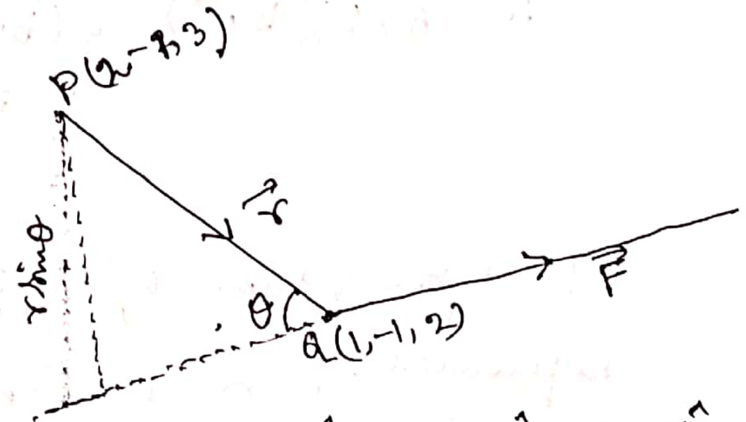
$\therefore$ volume of Parallelopiped $= 0$

$$|\vec{A}\cdot(\vec{B}\times\vec{C})| = 0$$

then $\alpha = ?$

Q: A force given by $\vec{F} = 3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point $(1,-1,2)$. Find the moment of $\vec{F}$ about the point $(2,-1,3)$.

Ans:- The force $\vec{F}$ acting at the point let $Q(1,-1,2)$. We will find the moment of $\vec{F}$ about the point $P(2,-1,3)$.



Now the vector from the point $P(2,-1,3)$ to $Q(1,-1,2)$ is $\vec{r} = (1-2)\hat{i}+(-1+1)\hat{j}+(2-3)\hat{k}$
$= -\hat{i}-\hat{k}$.

$\therefore$ The moment of the force $\vec{F}$ about $P(2,-1,3)$ is

$$\vec{M} = \vec{r}\times\vec{F}$$

$$= (-\hat{i}-\hat{k})\times(3\hat{i}+2\hat{j}-4\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= (0+2)\hat{i}-(4+3)\hat{j}+(-2-0)\hat{k}$$

$$= 2\hat{i}-7\hat{j}-2\hat{k}. \quad \text{Ans:}$$

Moment M = (magnitude of force $\vec{F}$)(perp. distance from P to the line of action of $\vec{F}$)
$= (|\vec{F}|)(r\sin\theta)$
$= F\,r\sin\theta$
$= |\vec{r}\times\vec{F}|$